An Extensible Framework for Modeling Simplicial and Cell Complexes

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Abstract

We introduce the Mangrove Topological Data Structure (Mangrove TDS) *framework for modeling simplicial and cell complexes. It is based on a graph-based representation of the data structures, called* mangrove*, which ensures an extensible description of any data structure without restrictions under a common application interface. Mangroves can be easily customized for any modeling need, including the efficient representation of non-manifold shapes, and of those cells, not directly encoded in a mangrove, that we call* ghost entities*. We discuss here the properties of this framework, and current and future developments.*

1. Introduction

Simplicial and cell complexes are extensively used to discretize digital shapes in many applications, including computer graphics, solid modeling, finite element analysis and simulation, scientific visualization, and geographic data processing. They allow modeling also *non-manifold shapes*, containing parts of different dimensions, and not necessarily embedded in the 3D Euclidean space. Informally, a manifold is a subset of the Euclidean space such that every neighborhood of each point is homeomorphic to an open ball.

In the literature, a large variety of topological data structures have been proposed for representing simplicial and cell complexes [DFH05]. They are formalized through *topological relations*, which capture the connectivity information of the cells in the complex they represent. In any case, many data structures are specific for geometry processing, or represent only 2D and 3D shapes. Hence, there is a need for a framework capable of supporting a wide number of representations under a common application interface. It should provide a common basis for performing quantitative comparisons, regarding performances of topological data structures. These latter can be implemented in heterogenous libraries, which do not allow performing coherent comparisons.

Moreover, this framework should provide a common platform for designing topological data structures efficiently without rewriting them from scratch. Specifically, it should be possible to customize a prototype of a topological data structure in order to satisfy any modeling need in the same spirit as the *rapid prototyping* techniques [Wri01]. Hence, this framework should be *extensible* and support a wide variety of representations, which are replaced dynamically in order to choose the most efficient one for a specific task.

Finally, it should support the efficient retrieval of topological queries on any representation, in particular when dealing with non-manifold shapes. In this context, it is interesting to detect non-manifold singularities efficiently. Note that recognizing whether a cell is a non-manifold singularity is decidable only for cell d-complexes, with $d < 6$ [Nab96].

2. The Mangrove TDS Framework

The connectivity information for the cells in a complex Γ can be described as a directed graph, which we call a *mangrove*. Recall that any topological data structure directly encodes a subset of topological relations, restricted to several cells in Γ, which are directly encoded. A node n_γ of a mangrove corresponds to any cell γ, while an arc $(n_\gamma, n_{\gamma'})$ describes a topological relation between cells γ and γ'. For each cell γ, only endpoints of arcs outgoing from n_γ are encoded. Any mangrove is *global* if it describes a topological data structure encoding all the cells in Γ, like the *Incidence Simplicial (IS)* data structure [DFHPC10], which also encodes a subset of immediate incidence relations. Otherwise, any mangrove is *partial*, and its nodes correspond to a subset of cells in Γ. For instance, the *Generalized Indexed data structure with Adjacencies (IA^*)* [CDFW11] is represented by a partial mangrove, since it encodes only vertices and top cells (those that do not bound other cells), plus a subset of adjacency relations, restricted to top cells.

We exploit mangroves as basis of our *Mangrove Topological Data Structure (Mangrove TDS)* framework in order to define a common prototype of any topological data structure. In fact, it is possible to define a mangrove $\mathcal{G}_\Gamma$, describing all cells and topological relations in Γ. This representation is verbose, but the graph-based representation of a specific data structure $\mathcal{M}$ is a subgraph of $\mathcal{G}_\Gamma$, which contains only nodes and arcs corresponding, respectively, to cells and topological relations directly encoded in $\mathcal{M}$. Thus, any topological data structure is simulated within our framework without introducing a relevant overhead.

Another contribution of this work is given by *ghost entities*, i.e., implicit representations of cells not encoded in any partial mangrove, like, for instance, the IA* data structure. Ghost entities are defined on simplicial and cell complexes, whose cells have a limited number of faces, like quad and unstructured hex meshes. The key idea consists of associating a cell with one top cell in its star in order to exploit adjacency relations, encoded only for top cells. The use of ghost entities improves the efficiency of topological queries and the expressive power of partial mangroves, which become equivalent to global mangroves [Can12]. In any case, ghost entities introduce a negligible overhead, independent of the size of a cell complex, thus the storage cost of partial mangrove remains almost unchanged. Finally, our implicit representations are suitable for applications in high dimensions, since they require a constant number of indices, which does not depend on the dimension of a complex.

We have implemented topological queries for extracting all the possible topological relations in the complex as well as for detecting non-manifold singularities for several data structures [Can12], including the IS and the IA* data structures. These two latter data structures support the efficient recognition of non-manifold singularities, according to our tests. The implementation of the common infrastructure of our framework, and of all the data structures, is contained in the *Mangrove Topological Data Structure (Mangrove TDS) Library* [CDF12], released under a GPL licence.

3. Discussion

Very few frameworks with the same properties of our Mangrove TDS framework have been proposed and implemented in the literature [Can12]. Most of them exploit a monolithic representation of a shape, which cannot be dynamically replaced, unless to completely rewrite or change the current software module [CGA11]. Internal representations of some frameworks [OVM12] are equivalent to data structures, like the *Incidence Graph* [Ede87], which do not efficiently support the recognition of non-manifold singularities. Finally, some frameworks can manipulate only 2- and 3-complexes, embedded in the Euclidean 3D space [OM02, OVM12].

Our current implementation of the Mangrove TDS Library [CDF12] does not support editing operators on mangroves. We are currently working in this direction. In particular, we are designing and implementing a new set of Euler operators on cell complexes [CDFI13], which preserve the simplicial homology, like Betti numbers and generators. These operations are useful to improve the efficiency of homology computations on simplicial and cell complexes.

We are also planning to exploit our framework for applications in high dimensions, due to the compactness of some representations, like the IA* data structure. For instance, our preliminary tests show that it may be up to 100 times more compact than the IS data structure when representing 8-dimensional shapes.

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