A Compact Representation for Topological Decompositions of Non-Manifold Shapes

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Introduction

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Each neighborhood of every point p is homeomorphic to one connected component of a ***ball***, centered at p.

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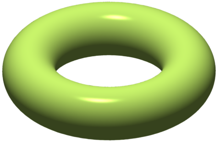

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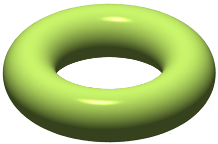

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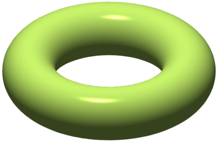

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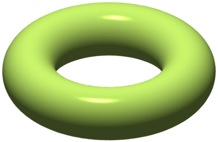

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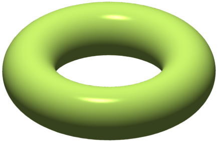

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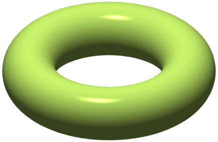

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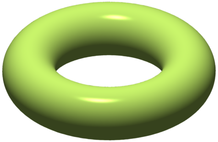

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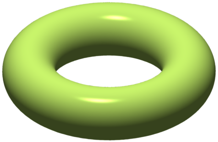

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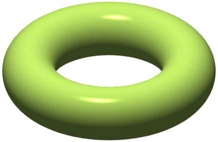

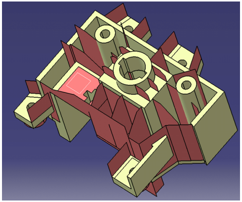

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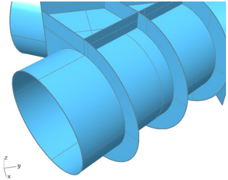

Representing Non-Manifold Shapes

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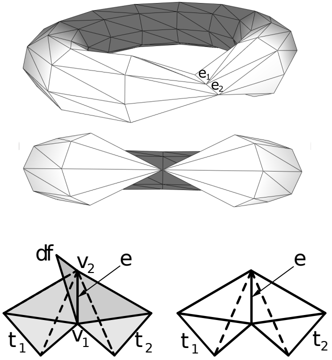

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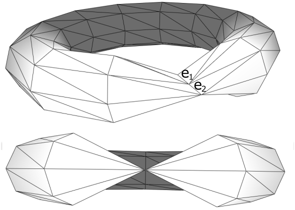

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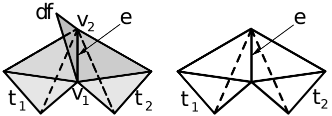

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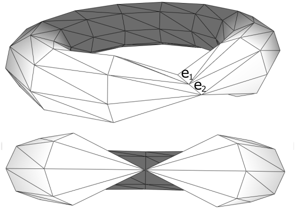

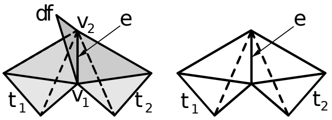

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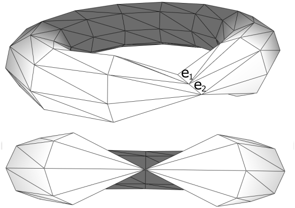

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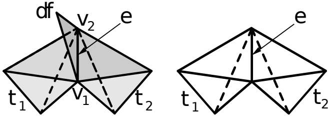

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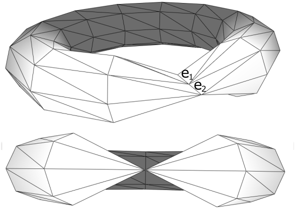

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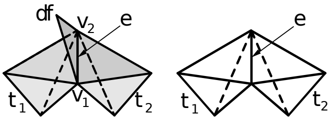

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Structural Model

connections among meaningful components (***global structure***)

Our Proposal: a Decomposition Approach

Main Property of Non-Manifold Shapes

Complex topology of a non-manifold shape offers ***valuable information*** for:

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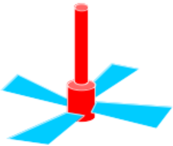

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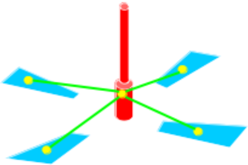

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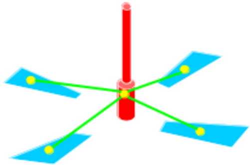

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Key Idea of our Approach

Expose explicitly and combine ***combinatorial*** and ***structural*** information

Compact Manifold-Connected (MC-) graph

Two-level graph-based representation of the ***MC-decomposition***, *Hui and De Floriani, 2007*

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Given a simplicial d-complex Σ and $k \leq d$:

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Does not ***bound*** any other simplex

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Sequence of top k-simplices in Σ, where each simplex is ***adjacent*** through a $(k-1)$-simplex, bounding ***at most two*** top k-simplices

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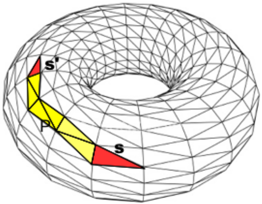

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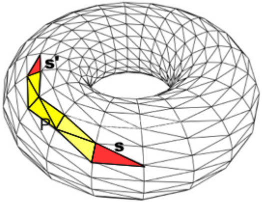

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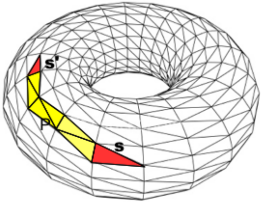

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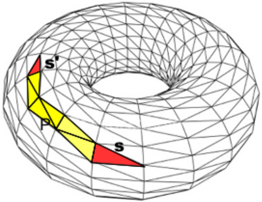

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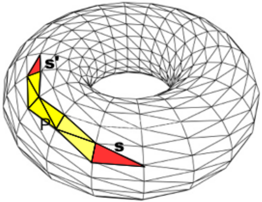

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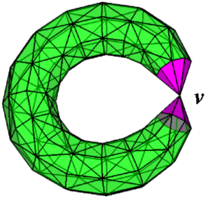

Superclass of manifolds, they may contain ***non-manifold singularities***

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- Decomposition of a simplicial complex Σ into its ***MC-complexes*** *(MC-components)*

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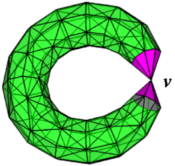

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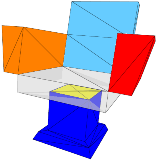

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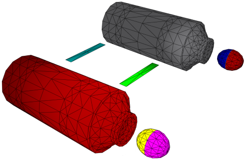

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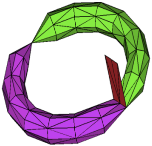

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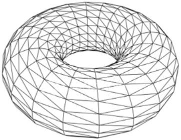

1 MC-component (for manifolds)

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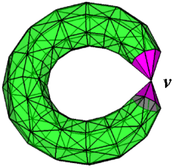

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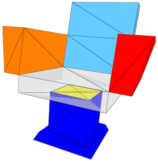

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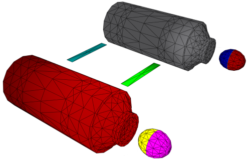

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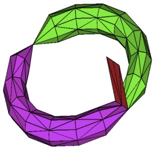
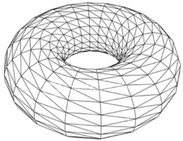


3 MC-components



1 MC-component (for manifolds)

MC-components' Intersection

Common subcomplex of some ***non-manifold singularities***

The Compact Manifold-Connected (MC-) graph

A two-level representation of the MC-decomposition, which integrates ***combinatorial*** and ***structural*** aspects.

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Lower Level (Combinatorial Aspects)

Describes a non-manifold shape by any topological data structure $\mathcal{M}_\Sigma$ (***unique***):

- the *Incidence Simplicial (IS)* data structure, *De Floriani et al., 2010*
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The Mangrove TDS Framework (Canino, 2012 - PhD. Thesis)

- Tool for the ***fast prototyping*** of topological data structures
- ***Extensible*** through dynamic plugins (***mangroves***)
- ***Any*** type of complexes is supported

The *Mangrove TDS Library* is released as GPL v3 software for the scientific community at
`http://mangrovetds.sourceforge.net`

The Compact MC-graph (Structural Aspects)

Describes the ***connectivity*** of MC-components by a hypergraph $\mathcal{G}_{\Sigma}^{\mathcal{C}} = (\mathcal{N}_{\Sigma}, \mathcal{A}_{\Sigma}^{\mathcal{C}})$

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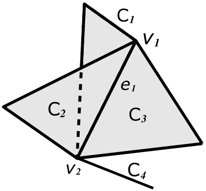

A hypernode in $\mathcal{N}_{\Sigma}$

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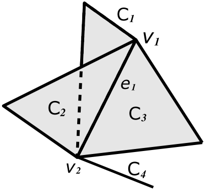

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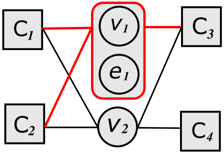

A hyperarc a in $\mathcal{A}^{\mathcal{C}}_{\Sigma}$

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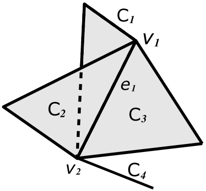

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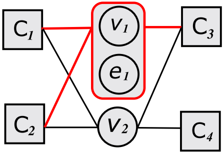

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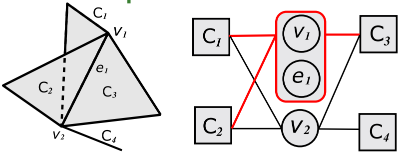
References are directed toward simplices in $\mathcal{M}_{\Sigma}$

Similar to a ***spatial index*** on any non-manifold shape

Storage Cost

$$S_C = n_C + \sum_{a \in \mathcal{A}_{\Sigma}^{C}} (k_a + s_a)$$

Other Representations of the MC-decomposition



Properties of our Compact MC-graph

- Few hyperarcs
- Minimizes duplications of intersections
- Maximal list of MC-components in hyperarcs

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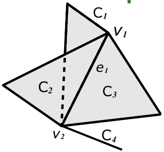

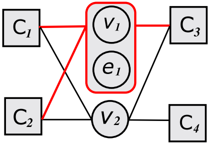

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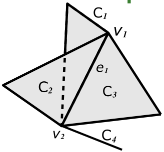

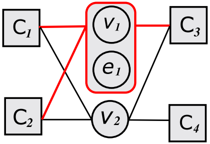

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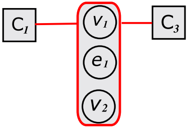
Arcs $\equiv$ ***intersections*** of only ***two*** MC-Components, formed by a subcomplex of non-manifold singularities



Verbose due to cliques

Less robust wrt to simmetry

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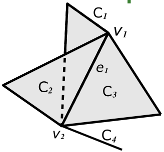

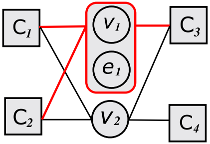

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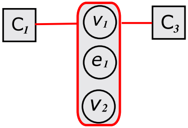

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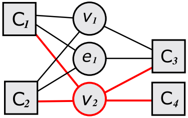
Exploded MC-Graph (Canino and De Floriani, 2011)

A hyper-arc $\equiv$ one ***non-manifold singularity*** σ, and connects all the MC-components, bounded by σ



Too much hyperarcs

Many duplications of the same MC-components in hyperarcs

Experimental Results (with our Mangrove TDS Library)

Digital shapes are freely available from `http://indy.disi.unige.it/nmcollection`

2D shapes (Storage cost and Properties of MC-graphs)

Shape	n_C	a_E	a_P	a_C	S_E	S_P	S_C
Carter	45	641	79	48	$3.8k$	$2.6k$	$1.2k$
Chandelier	130	616	328	96	$2.6k$	$2.6k$	$1k$
Pinched Pie	120	$1.4k$	$1.4k$	192	$4.8k$	$9.6k$	$1.9k$
Tower	169	$1.4k$	$13k$	165	$5.9k$	$43k$	$2.1k$

n_C: #MC-components
a_E, a_P, a_C: #(hyper)arcs
S_E, S_P, S_C: storage costs

For 2D shapes:

$a_E \approx 8.9 \times a_C$, $a_P \approx 23 \times a_C$
$S_E \approx 2.8 \times S_C$, $S_P \approx 7.7 \times S_C$

3D shapes (Storage cost and Properties of MC-graphs)

Shape	n_C	a_E	a_P	a_C	S_E	S_P	S_C
Chime	27	29	47	28	133	210	127
Flasks	8	76	10	6	300	232	98
Teapot	$2.9k$	$1.2k$	$18.1k$	$1k$	$10.4k$	$57.5k$	$10.1k$
Wheel	115	136	520	88	675	$1.7k$	563

For 3D shapes:

$a_E \approx 4.1 \times a_C$, $a_P \approx 6.8 \times a_C$
$S_E \approx 1.6 \times S_C$, $S_P \approx 3.2 \times S_C$

Our experimental results confirm properties of the Compact MC-graph

Experimental Results (cont'd)

Comparisons with the *Incidence Graph*, *Edelsbrunner, 1987*

Shape	S_C	S_{IA*}	S_{IG}	$S_C + S_{IA*}$
Carter	$1.2k$	$52k$	$95k$	$53.2k$
Chandelier	$1k$	$120k$	$220k$	$121k$
Tower	$2.1k$	$124k$	$221k$	$126.1k$
Flasks	98	$29k$	$104k$	$29.1k$
Teapot	$10.1k$	$85k$	$220k$	$95.1k$
Sierpinski 3D	$458k$	$524k$	$3.67M$	$0.98M$
Sierpinski 4D	$664k$	$781k$	$11.6M$	$1.44M$
Sierpinski 5D	$467k$	$559.6k$	$7.7M$	$1M$

Combined with the IA* data structure,
Canino et al., 2011

S_{IA*} : storage cost of the IA*
S_{IG}: storage cost of the IG

For 2D shapes: $S_{IG} \approx 1.45 \times S_{IA*}$
For 3D shapes: $S_{IG} \approx 3.2 \times S_{IA*}$
For 4D shapes: $S_{IG} \approx 8 \times S_{IA*}$
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Interesting result (wrt the Incidence Graph)

The *Compact MC-graph*, combined with the *IA** data structure, is ***more compact*** than the incidence graph:

- our contribution is a ***structural model*** (topological + structural aspects)
- the IG data structure is a topological data structure (***local connectivity***)

Conclusions and Future Work

Compact Manifold-Connected (MC-) graph

Two-level graph-based representation of the

MC-decomposition, *Hui and De Floriani, 2007*

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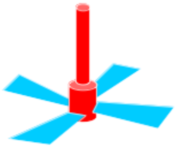
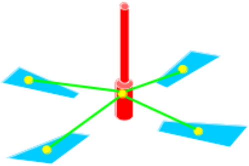


Topological data structure
(Local Connectivity)



Structural model
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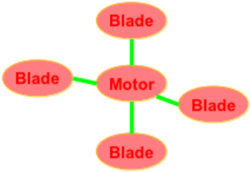

Semantic model
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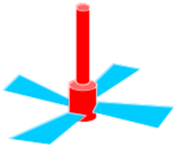

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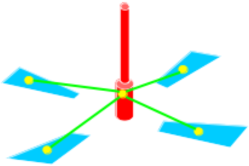

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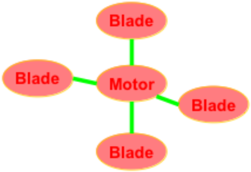

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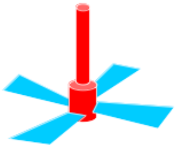

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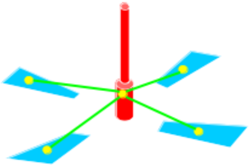

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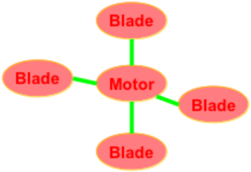

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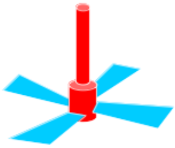

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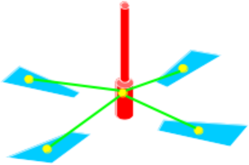

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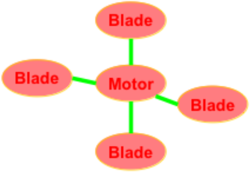

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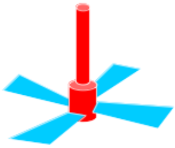

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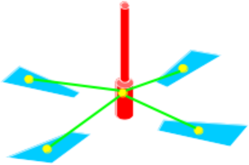

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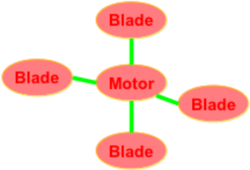

Semantic model (Future Work)

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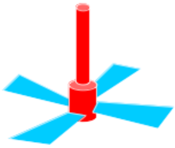

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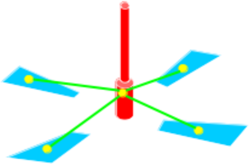

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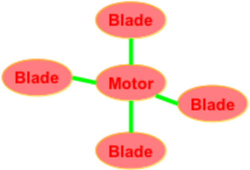

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Acknowledgements

We thank:

- anonymous reviewers for their useful suggestions
- the Italian Ministry of Education and Research (the PRIN 2009 program)
- the National Science Foundation (contract IIS-1116747)

These slides are available on

`http://www.disi.unige.it/person/CaninoD`

Thank you much for your attention!

Interesting Papers and References

- M. Attene, D. Giorgi, M. Ferri, and B. Falcidieno, *On Converting Sets of Tetrahedra to Combinatorial and PL Manifolds*, Computer-Aided Design, 26(8):850-864, Elsevier Press, 2009
- M. Botsch, L. Kobbelt, M. Pauly, P. Alliez, B. Lévy, *Polygon Mesh Processing*, CRC Press, 2010
- D. Boltcheva, D. Canino, S. Merino, J.-C. Léon, L. De Floriani, F. Hétroy, *An Iterative Algorithm for Homology Computation on Simplicial Shapes*, Computer-Aided Design, 43(11):1457-1467, Elsevier Press, SIAM Conference on Geometric and Physical Modeling (GD/SPM 2011)
- D. Canino, L. De Floriani, *A Decomposition-based Approach to Modeling and Understanding Arbitrary Shapes*, 9th Eurographics Italian Chapter Conference, Eurographics Association, 2011
- D. Canino, L. De Floriani, K. Weiss, *IA*: An Adjacency-Based Representation for Non-Manifold Simplicial Shapes in Arbitrary Dimensions*, Computer & Graphics, 35(3):747-753, Elsevier Press, Shape Modeling International 2011 (SMI 2011), Poster
- L. De Floriani and A. Hui, *Data Structures for Simplicial Complexes: an Analysis and a Comparison*, In Proceedings of the 3rd Eurographics Symposium on Geometry Processing (SGP '05), pages 119-128, ACM Press, 2005
- L. De Floriani, A. Hui, D. Panozzo, D. Canino, *A Dimension-Independent Data Structure for Simplicial Complexes*, In S. Shontz Ed., Proceedings of the 19th International Meshing Roundtable, pages 403-420, Springer, 2010
- L. De Floriani, M. Mesmoudi, F. Morando, and E. Puppo, *Decomposing Non-manifold Objects in Arbitrary Dimension*, Graphical Models, 65:1/3:2-22, Elsevier Press, 2003
- H. Desaulniers and N. Stewart, *An Extension of Manifold Boundary Representations to the r-sets*, ACM Transactions on Graphics, 11(1):40-60, 1992
- H. Edelsbrunner, *Algorithms in Combinatorial Geometry*, Springer, 1987

Interesting Papers and References (cont'd)

- A. Hui and L. De Floriani, *A Two-level Topological Decomposition for Non-Manifold Simplicial Shapes*, In Proceedings of the ACM Symposium on Solid and Physical Modeling, pages 355-360, ACM Press, 2007
- A. Nabutovsky, *Geometry of the Space of Triangulations of a Compact Manifold*, Communications in Mathematical Physics, 181:303-330, 1996
- S. Pesco, G. Tavares, H. Lopes, *A Stratification Approach for Modeling Two-dimensional Cell Complexes*, Computer & Graphics, 28:235-247, 2004
- J. Rossignac and M. O'Connor, *A Dimension-independent for Point-sets with Internal Structures and Incomplete Boundaries*, Geometric Modeling for Product Engineering, North-Holland, 1989
- J. Rossignac and D. Cardoze, *Matchmaker: manifold BReps for Non-manifold R-sets*, Proceedings of the ACM Symposium on Solid Modeling and Applications, ACM Press, pages 31-41, 1999