An Extensible Framework for Modeling Simplicial and Cell Complexes

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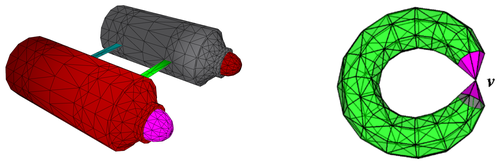
Motivations

- Need to represent ***simplicial*** and ***cell complexes*** of any dimension and kind.
- Retrieving ***topological information efficiently*.**



- Several ***data structures*** have been developed in the literature.
- A tool for their ***fast design and implementation*** under a common application interface lacks:
  - ***avoiding*** to design and implement topological data structures from scratch;
  - performing ***coherent comparisons*** regarding their performance.

Key Idea

- A ***common representation*** of any data structure ***(prototype),*** customized to ***simulate*** a specific representation ***without a relevant overhead.***
- Common prototype is dynamically ***replaced at run-time***, if necessary (***plugins***) in order to:
  - exploit the ***most suitable representation*** wrt the storage cost and the efficiency of queries;
  - ***hide*** internal details of the tool and guarantee a ***short learning curve***.

The Mangrove Topological Data Structure (Mangrove TDS) Framework

- ***Rapid prototyping*** of topological data structures for ***cell*** and ***simplicial complexes***, that are described by graph-based representations, called ***mangroves***;
- ***Implicit representations*** of cells through ***ghost entities***.

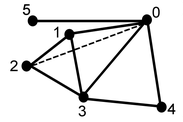
Mangroves

- ***Graph-based representation*** of data structures:
  - ***nodes: simplices*** and ***cells***, directly encoded in the data structure;
  - ***arcs***: ***topological relations,*** directly encoded in the data structure.
- A topological data structure is a ***subgraph*** of the mangrove representing ***all*** cells and topological relations in the cell complex.

Ghost Entities

- A p-cell γ may be either a ***top p-cell***, or a ***p-face*** of any top *t-cell* γ' in its star (with $p \leq t$).
- Represented as *(t,i,p,j)*, where *i* is the unique identifier of γ', and *j* is the unique identifier of γ as *p-face* of γ'.
- ***Not unique*** and ***independent*** of any specific enumeration of faces and top cells.
- Suitable for ***high dimensions*** (always four values) unlike their ***explicit representation*.**
- Make topological queries ***3X faster*** in the IA* data structure, *Canino et all., 2011*, wrt the IS data structure, *De Floriani et all., 2010.*

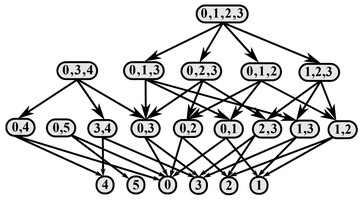
Example #1: IS data structure



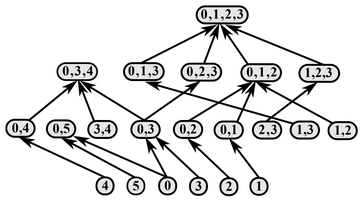
- ***Abstract simplicial*** and ***cell complexes*** of any dimension.
- Encodes ***all*** cells.

For ***each*** p-cell γ:

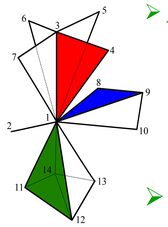
- boundary relation $R_{p,p-1}(\gamma)$, formed by *(p-1)-faces* of γ;



- partial co-boundary relation $R^*_{p,p+1}(\gamma)$, formed by one ***arbitrary (p+1)-cell*** for each component in the link of γ.



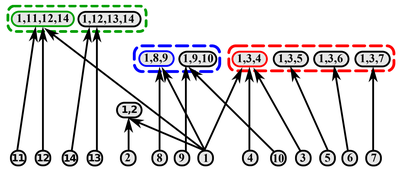
Example #2: IA* data structure



- ***Abstract simplicial*** and a specific class of ***cell complexes*** (e.g., quad and hex meshes) of any dimension.
- Encodes ***vertices*** and ***top*** cells in a complex.

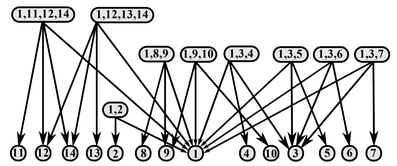
For each vertex v:

- partial co-boundary relation $R^*_{0,p}(v)$, formed by one ***arbitrary top p-cell*** for each ***(p-1)-connected component*** in the star of v.

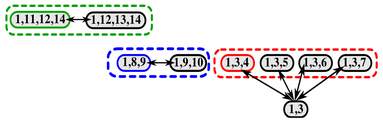


For ***each*** top p-cell γ:

- boundary relation $R_{p,0}(\gamma)$, formed by ***vertices*** on the boundary of γ;



- adjacency relation $R^*_{p,p}(\gamma)$, formed by ***top p-cells*** adjacent to γ (if $p>1$).
- ***Compact encoding*** of $R^*_{p,p}(\gamma)$ through relation $R^*_{p-1,p}(\tau)$, formed by all top *p-cells* incident at a *(p-1)-face* τ of γ:
  - if τ is on the boundary of ***more than two*** top *p-cells*, then $R^*_{p-1,p}(\tau)$ is stored once and $R^*_{p,p}(\gamma)=R^*_{p-1,p}(\tau)$.



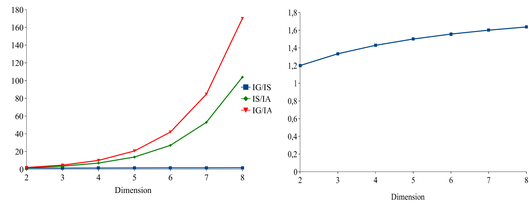
Implementation

- The ***Mangrove TDS Library*** is a C++ tool containing the complete ***implementations*** of our framework, and of ***six data structures***, including the IS and IA* data structures.
- Based on ***templated programming*** techniques and completely ***platform-independent***.
- Exploits an ***array-based*** storage with ***safe iterators*** and ***garbage collector*** mechanism for each collection of cells.
- Possibility of ***dynamically associate*** any type of information (***properties***) with cells, also with ***ghost entities***.
- ***Publicly released*** under GPL3, visit:

http://mangrovetds.sourceforge.net

Current and Future Work

- ***Editing operators*** on simplicial and cell complexes, e.g., ***homology preserving*** and ***modifying operators***, like stellar operators and those in *Comic et all., 2013.*
- Applications in ***high dimensions***, since the IA* data structure may be ***very compact***.



Comparisons with Other Frameworks

- Existing tools exploit a ***fixed representation***, that cannot be easily replaced, thus it is ***not flexible***.
- Internal representations are usually ***equivalent*** to the *Incidence Graph* and to extensions of the *Half-Edge data structure.*
- This means large ***overhead*** for manifolds and ***not efficient*** identification of non-manifold singularities.

	OpenMesh	*OpenVolumeMesh*	*VCGLib*	*CGAL*	*Mangrove TDS*
Type of Complexes	Cell	Cell	Simplicial	Any	Any
Dimension of Complexes	Up to 2	Up to 3	Up to 3	Any	Any
Internal Representation	Incidence-based	Incidence-based	Adjacency-based	Several	Any
Flexible Representation	No	No	No	Yes (modules)	Yes (plugins)
Identification of Non-Manifold Singularities	Only at vertices	Not efficient	Complete	Complete	Efficient

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