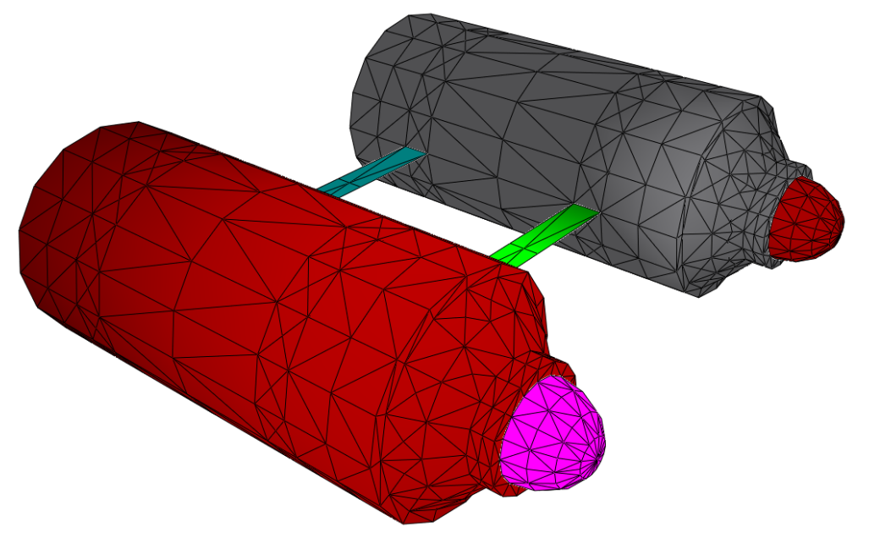
An Extensible Framework for Modeling Simplicial Complexes

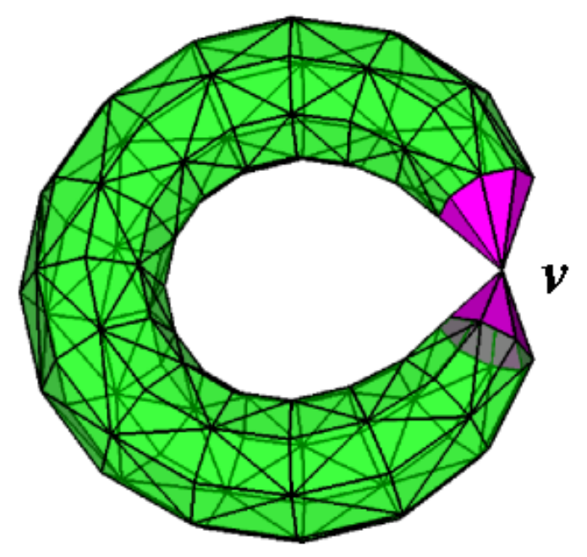
David CANINO Leila DE FLORIANI

University of Genova (Italy) and University of Maryland at College Park, MD (USA)

Problem

- Need to represent ***simplicial shapes*** of any dimension and with a ***complex topology***:
  - retrieving ***topological information***;
  - identifying ***non-manifold singularities***.





- Several ***data structures*** have been developed in the literature *[DFH05]*, also for ***non-manifolds***, but they are optimized and restricted to a ***specific task.***
- A common framework for their ***fast prototyping*** is currently lacking, but it would be ***interesting*** and ***suitable*** for many applications.

Key Idea

- A framework for this task must satisfy (at least) the following ***design choices*** *[SB11]:*
  - ***flexibility:*** common representation of any data structure, which can be ***dynamically replaced*** and ***customized*** at run-time, if necessary (***plugins***);
  - ***efficiency:*** exploit and choose the ***most suitable representation*** wrt any application need:
    - ***time efficiency*** for topological queries and ***restricted storage cost***;
    - ***expressive power*** wrt encoded information;
  - ***easy-to-use:*** hide internal details and require a ***short learning curve*** wrt other tools in the literature.

Comparisons with other tools

- Most of other tools exploit a ***fixed representation***, which cannot be easily replaced, thus they are ***not flexible***.
- The internal representations of some tools are ***equivalent*** to the *Incidence Graph [Ede87]* and to the *Half-Edge (HE) data structure [Man88]* (see *[Can12]* for a complete analysis).
- The *HE data structure* and its 3D extensions are ***restricted*** to the representations of ***manifolds***.
- The *Incidence Graph* exhibits a ***large overhead*** for manifolds, and does ***not allow*** for the ***efficient identification*** of non-manifold simplices *[DFH05]*.

	OpenMesh	*OpenVolumeMesh*	*VCGLib*	*CGAL*	*Mangrove TDS*
Type of Complexes	Cell	Cell	Simplicial	Any	Simplicial
Dimension of Complexes	Up to 2	Up to 3	Up to 3	Any	Any
Internal Representation	Incidence-based	Incidence-based	Adjacency-based	Several	Any
Flexible Representation	No	No	No	Yes (modules)	Yes (plugins)

The Mangrove Topological Data Structure (Mangrove TDS) framework

- We propose the ***Mangrove Topological Data Structure (Mangrove TDS)*** framework:
  - ***fast prototyping*** of topological data structures for simplicial complexes, described as ***mangroves;***
  - ***implicit representations*** of simplices, not encoded in a mangrove, called ***ghost simplices***;
  - ***completely satisfies*** all the design choices proposed in *[SB11].*

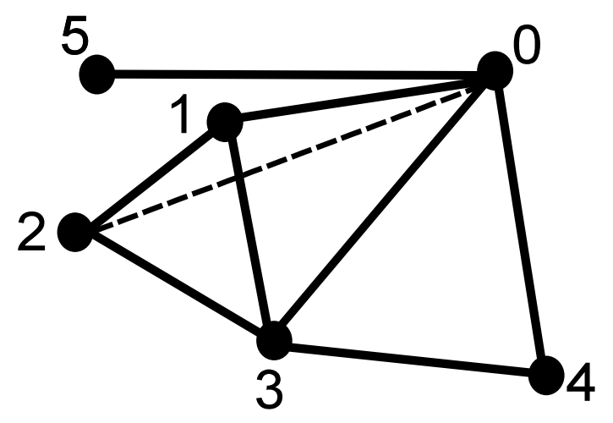
Mangroves

- ***Graph-based representations*** of data structures:
  - ***nodes*** correspond to ***simplices***, encoded directly in any data structure (possibly not all the simplices);
  - ***arcs*** correspond to ***topological relations,*** restricted to the subset of simplices directly encoded.
- They can represent ***any*** topological data structure, also for ***non-manifolds***, without restrictions wrt their dimension and the embedding Euclidean space.
- They can be ***easily customized*** for any modeling need, and dynamically loaded (***plugin***).
- They can encode either all the simplices (***global***), or only a subset (***restricted***).

Ghost Simplices

- A p-simplex σ may be either any ***top p-simplex***, or a ***p-face*** of any top k-simplex σ', and is represented as a 4-tuple (k,i,p,j), where i is the unique identifier of σ', and j is the unique identifier of σ as a p-face of σ'.
- Suitable to represent simplices in ***high dimensions*** (always four values), instead of a variable list of vertices (***explicit representation*** of a simplex).
- Make queries ***3X faster*** for any restricted mangrove, e.g., the IA* data structure *[CDFW11]*, wrt any global mangrove, e.g., the IS data structure *[DFHPC10]*

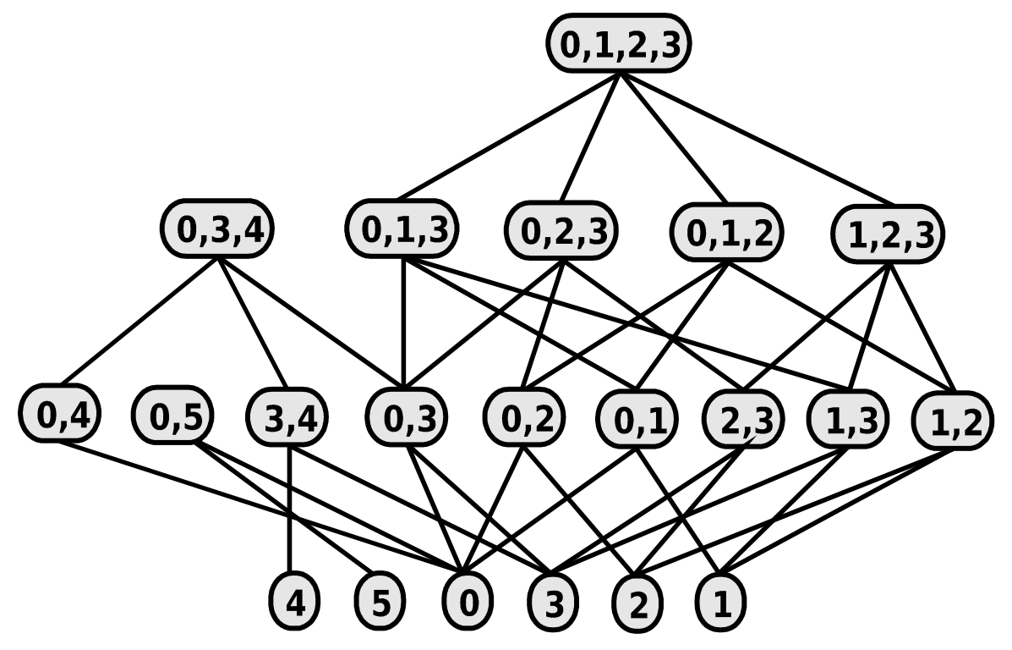
Example #1: IS data structure



- Represents ***abstract simplicial complexes*** of any dimension.
- Encodes ***all*** simplices.

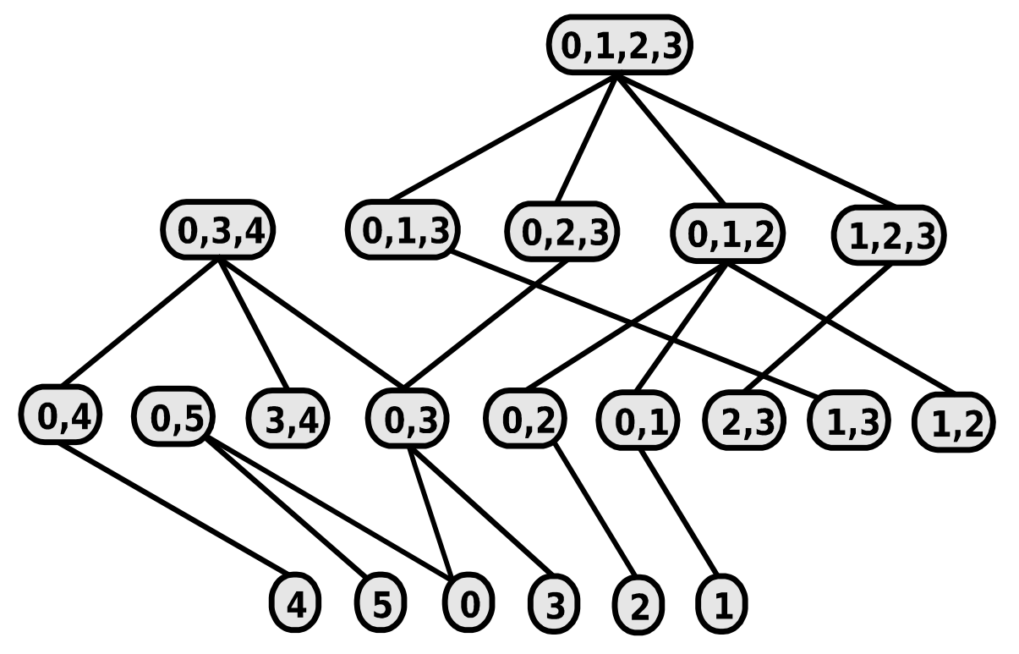
For ***each*** p-simplex σ:

- boundary relation $R_{p,p-1}(\sigma)$, formed by $(p-1)$-simplices on the boundary of σ;



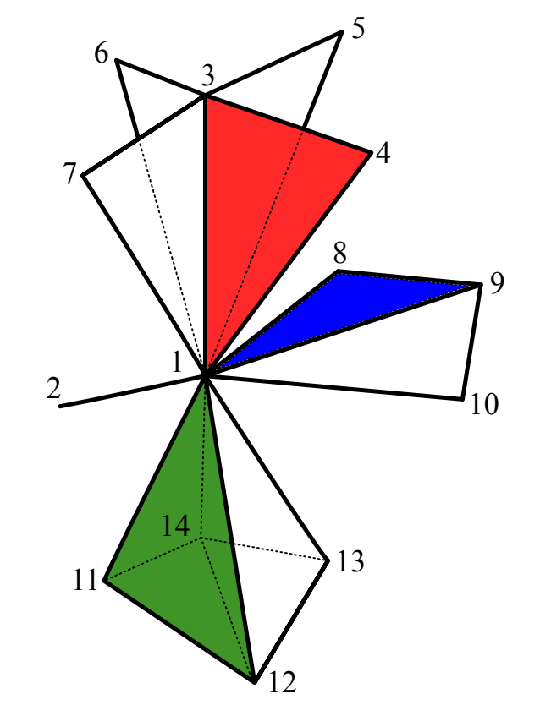
$R_{2,1}(0,3,4)=\{(0,4);(0,3);(3,4)\}$

- partial co-boundary relation $R^*_{p,p+1}(\sigma)$, formed by one ***arbitrary (p+1)-simplex*** for each component in the link of σ.



$R^*_{0,1}(0)=\{(0,3);(0,5)\}$

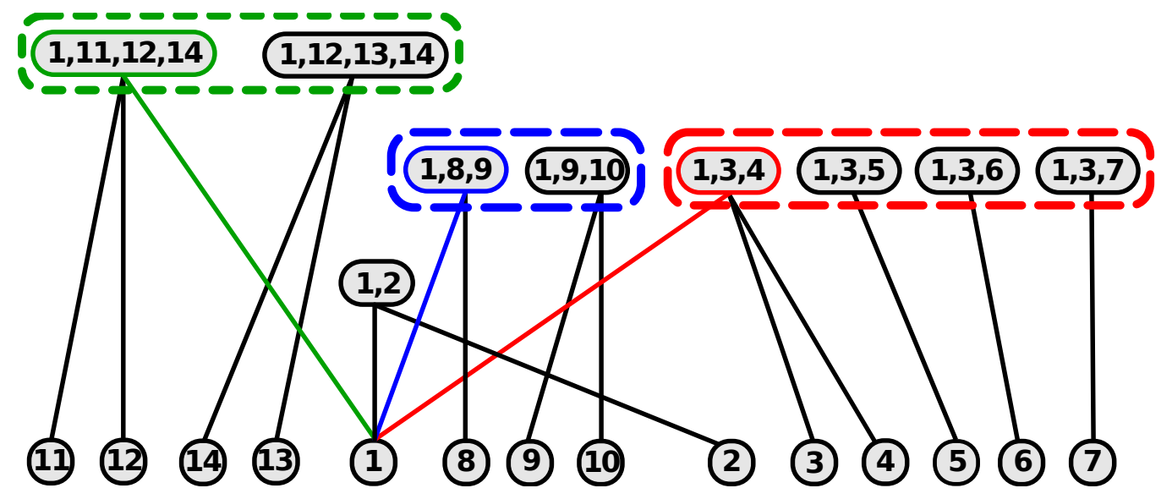
Example #2: IA* data structure



- Represents ***abstract simplicial complexes*** of any dimension.
- Encodes ***vertices*** and ***top*** simplices.

For each vertex v:

- partial co-boundary relation $R^*_{0,p}(v)$: one ***arbitrary top p-simplex*** for each ***(p-1)-connected component*** in the star of v.

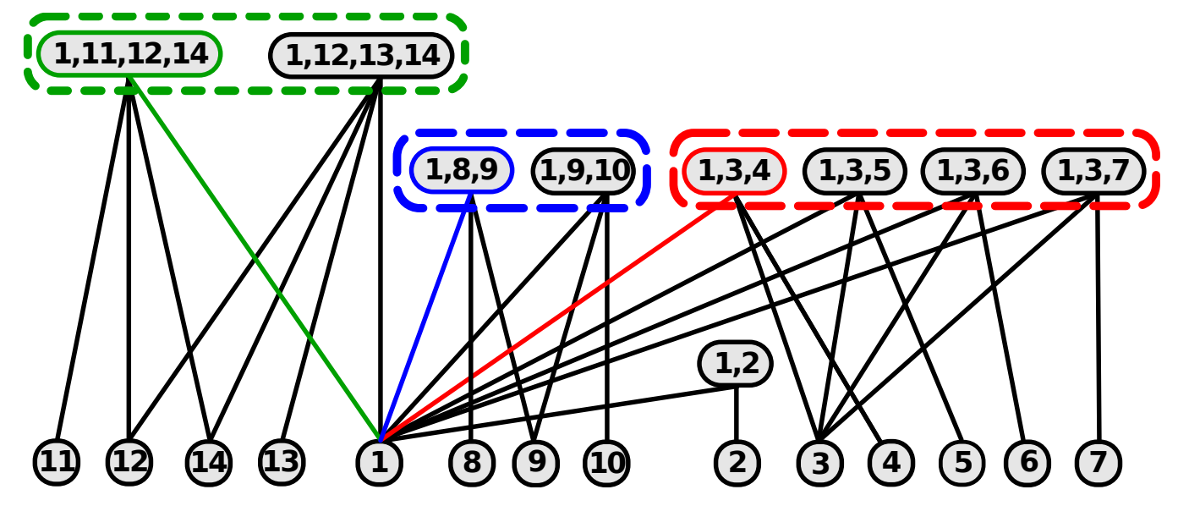


$R^*_{0,1}(1)=\{(1,2)\}$ - $R^*_{0,2}(1)=\{(1,3,4);(1,8,9)\}$

$R^*_{0,3}(1)=\{(1,11,12,14)\}$

For ***each*** top p-simplex σ:

- boundary relation $R_{p,0}(\sigma)$, formed by $p+1$ vertices on the boundary of σ;



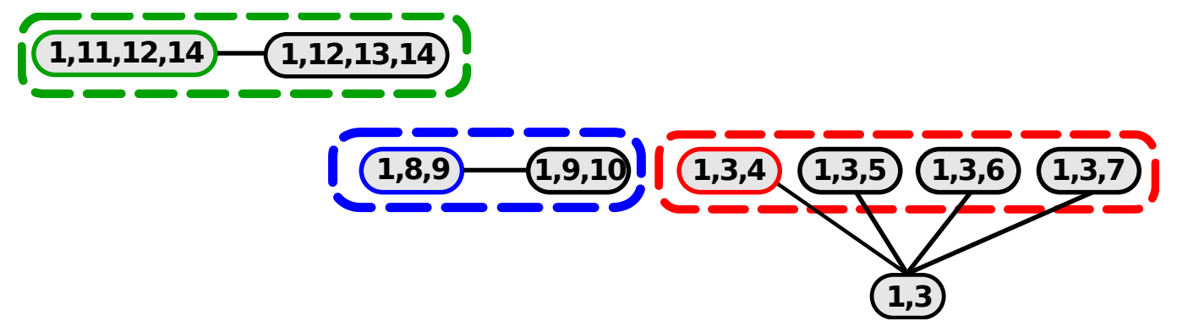
$R_{1,0}(1,2)=\{1,2\}$ - $R_{2,0}(1,3,4)=\{1,3,4\}$, $R_{2,0}(1,8,9)=\{1,8,9\}$

$R_{3,0}(1,11,12,14)=\{1,11,12,14\}$

- adjacency relation $R^*_{p,p}(\sigma)$, if $p>1$, formed by the ***top p-simplices*** adjacent to σ.

Adjacency relation $\boldsymbol{R^*_{p,p}(\sigma)}$ along a *(p-1)-face* τ of σ can be ***simplified*** through relation $R^*_{p-1,p}(\tau)$, formed by top p-simplices incident at τ:

- if τ is on the boundary of ***more than two*** top p-simplices, then $R^*_{p-1,p}(\tau)$ is stored only once (***non-manifold adjacency***);
- otherwise, a top p-simplex, adjacent to σ, is stored (***manifold adjacency***).



Manifold adjacency along edge *(1,9)* and triangle *(1,12,14)*.

Non-manifold adjacency along edge *(1,3)*.

Implementation

- The ***Mangrove TDS Library*** is a C++ tool, which contains the complete implementation of our framework, plus of ***six*** data structures, including the IS and the IA* data structures.
- It is based on ***templated programming*** techniques, and is completely ***multi-platform***.
- It exploits an ***array-based*** storage with ***iterators*** and ***garbage collector*** for each collections of simplices.
- It is possible to ***dynamically*** associate ***properties*** with any simplex, including with ***ghost simplices***.
- It is publicly released under ***GPL*** version 3, visit: ***http://mangrovetds.sourceforge.net***
- Our tests show that the IS and IA* data structures are ***effective*** representations for ***non-manifolds*** wrt their storage cost, identification of non-manifold simplices, and efficiency of queries.

Current and Future work

- Extensions of the IS and the IA* data structures for ***quad*** and unstructured ***hexahedral meshes***.
- Extensions to ***cell complexes*** (the Incidence Graph is already in the ***Mangrove TDS Library*** *[Can12]*).
- ***Editing operators*** on simplicial and cell complexes: ***homology*** preserving and modifying operators.

References

[Can12] CANINO D., *Tools for Modeling and Analysis of Non-Manifold Shapes*, PhD. Thesis, Department of Computer Science, University of Genova, Genova, Italy (2012)

[CDFW11] CANINO D., DE FLORIANI L., WEISS K., *IA*: an Adjacency-based Representation for Non-Manifold Simplicial Shapes in Arbitrary Dimensions*, Computer & Graphics – SMI special issue (2011)

[CGAL] The *CGAL LIBRARY* - *http://www.cgal.org*

[DFH05] DE FLORIANI L., HUI A.*, Data Structures for Simplicial Complexes: an Analysis and a Comparison*, Symposium on Geometric Processing (2005)

[DFHPC10] DE FLORIANI L., HUI A., PANOZZO D., CANINO D., *A Dimension-Independent Data Structure for Simplicial Complexes*, 19th International Meshing Roundtable (2010)

[Ede87] H. EDELSBRUNNER, *Algorithms in Combinatorial Geometry*, Springer (1987)

[Man88] M. MANTYLA, *An Introduction to Solid Modeling*, Comp. Sci. Press (1988)

[OpenMesh] The *OPEN MESH LIBRARY* - *http://www.openmesh.org*

[OpenVolumeMesh] The *OPEN VOLUME MESH LIBRARY - http://www.openvolumemesh.org*

[SB11] SIEGER D., BOTSCH M., *Design, Implementation, and Evaluation of the Surface_Mesh Data Structure*, 20th International Meshing Roundtable (2011)

[VCGLib] The *VCG LIBRARY* - *http://vcg.sourceforge.net*

Funded by Italian Ministry of Education and Research under the PRIN 2009 Program and by National Science Foundation under grant number IIS-1116747.